Valuing Non-Interest-Bearing Liabilities
Growth Rate is Time-Dependent

Gary Schurman MBE, CFA

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In this white paper we will extend the model in the previous white paper (notional growth rate is constant) and assume that the notional growth rate is time-dependent (i.e. mean-reverting). To that end we will work through the following hypothetical problem...

Our Hypothetical Problem

We are currently standing at time zero and are tasked with determining the market value of a ABC Company's non-interest-bearing liabilities. Our go-forward model assumptions are...

Description	Value
Notional value at time zero ($)	1,000,000
Annual short-term notional value growth rate (%)	18.50
Annual long-term notional value growth rate (%)	3.50
Annual investment yield (%)	4.75
Annual risk-adjusted discount rate (%)	9.50
Income tax rate (%)	15.50
Ratio of NIBL to notional value (%)	20.00
Transition half-life in years (#)	4.25

Note: Notional value is defined as tangible assets for banks and annualized revenue for non-banks.

Question 1: What is the book value of non-interest-bearing liabilities at time zero?

Question 2: What is the market value of non-interest-bearing liabilities at time zero?

Building Our Model

We will define the variable λ to be the rate of mean reversion, which is the rate at which the short-term rate transitions to the long-term rate over time. The equation for the rate of mean reversion is... [1]

$$\lambda = -\frac{\ln(0.50)}{T} \text{ ...where... } T = \text{Transition half-life in years} \tag{1}$$

Using Equation (1) above and our go-forward model assumptions above, the rate of mean reversion for our problem is...

$$\lambda = -\frac{\ln(0.50)}{1.25} = 0.1631 \tag{2}$$

We will define the variable ω_S to be the continuous-time, short-term notional growth rate, the variable ω_L to be the continuous-time, long-term notional growth rate, and the variable ω_t to be the continuous-time notional growth rate at time t. Using Equation (1) above, the equation for the notional growth rate at time t is...

$$\omega_t = \omega_L + \Delta(\omega) \operatorname{Exp}\left\{ -\lambda\, t \right\} \text{ ...where... } \Delta(\omega) = \omega_S - \omega_L \tag{3}$$

We will define the variable Γ_t to be the cumulative notional growth rate over the time interval $[0, t]$. Using Equation (3) above, the equation for the cumulative notional growth rate at time t is... [1]

$$\Gamma_t = \int_0^t \omega_s \, \delta s = \frac{\Delta(\omega)}{\lambda} + \omega_L \, t - \frac{\Delta(\omega)}{\lambda} \operatorname{Exp} \left\{ -\lambda \, t \right\} \tag{4}$$

We will define the variable N_t to be notional value at time t. Notional value is defined as tangible assets for banks and annualized operating revenue for non-banks. Using Equation (4) above, the equation for notional value at time t is...

$$N_t = N_0 \operatorname{Exp} \left\{ \Gamma_t \right\} = N_0 \operatorname{Exp} \left\{ \frac{\Delta(\omega)}{\lambda} + \omega_L \, t - \frac{\Delta(\omega)}{\lambda} \operatorname{Exp} \left\{ -\lambda \, t \right\} \right\} \tag{5}$$

We will define the variable L_t to be non-interest-bearing liabilities at time t and the variable η to be the ratio of non-interest-bearing liabilities to notional value. Using Equation (5) above, the equation for non-interest-bearing liabilities at time t is...

$$L_t = \eta \, N_t = \eta \, N_0 \operatorname{Exp} \left\{ \frac{\Delta(\omega)}{\lambda} + \omega_L \, t - \frac{\Delta(\omega)}{\lambda} \operatorname{Exp} \left\{ -\lambda \, t \right\} \right\} \tag{6}$$

We will define the variable I_t to be annualized after-tax investment income at time t, the variable α to be the continuous-time, pre-tax investment yield, and the variable τ to be the income tax rate. Using Equation (6) above, the equation for annualized after-tax investment income at time t is...

$$I_t = \alpha \, (1 - \tau) \, L_t = \alpha \, (1 - \tau) \, \eta \, N_0 \operatorname{Exp} \left\{ \frac{\Delta(\omega)}{\lambda} + \omega_L \, t - \frac{\Delta(\omega)}{\lambda} \operatorname{Exp} \left\{ -\lambda \, t \right\} \right\} \tag{7}$$

We will define the variable $I_{m,n}$ to be after-tax investment income recognized over the time interval $[m, n]$. Using Equation (7) above, the equation for cumulative after-tax investment income is...

$$I_{m,n} = \int_m^n I_t \, \delta t = \alpha \, (1 - \tau) \, \eta \, N_0 \int_m^n \operatorname{Exp} \left\{ \frac{\Delta(\omega)}{\lambda} + \omega_L \, t - \frac{\Delta(\omega)}{\lambda} \operatorname{Exp} \left\{ -\lambda \, t \right\} \right\} \delta t \tag{8}$$

Note that there is no closed-form solution to Equation (8) above and therefore must be solved via numerical integration.

We will define the variable V_0 to be the present value at time zero of after-tax investment income over the time interval $[0, \infty]$ and the variable κ to be the continuous-time, risk-adjusted discount rate. Using Equation (8) above, the equation for the present value of after-tax investment income is...

$$V_0 = \int_0^\infty I_t \operatorname{Exp} \left\{ -\kappa \, t \right\} \delta t = \alpha \, (1 - \tau) \, \eta \, N_0 \int_0^\infty \operatorname{Exp} \left\{ \frac{\Delta(\omega)}{\lambda} + (\omega_L - \kappa) \, t - \frac{\Delta(\omega)}{\lambda} \operatorname{Exp} \left\{ -\lambda \, t \right\} \right\} \delta t \tag{9}$$

Note that the constraint in Equation (9) above is that $\omega_L < \kappa$.

Answers To Our Hypothetical Problem

Using the go-forward model assumptions above, the parameters to our problem are...

Symbol	Equation	Value
N_0	Equation (5)	1,000,000
α	$\ln(1 + 0.0475)$	0.0464
κ	$\ln(1 + 0.0950)$	0.0908
ω_S	$\ln(1 + 0.1850)$	0.1697
ω_L	$\ln(1 + 0.0350)$	0.0344
$\Delta(\omega)$	$\omega_S - \omega_L$	0.1353
λ	Equation (2)	0.1631
η	NA	0.2000
τ	NA	0.1550

Question 1: What is the book value of non-interest-bearing liabilities at time zero?

Using Equation (6) above and the go-forward model assumptions above, the answer to the question is...

$$\text{Book value at time zero} = L_0 = 0.20 \times 1,000,000 = 200,000 \tag{10}$$

Question 2: What is the market value of non-interest-bearing liabilities at time zero?

Using Equation (9) above and our go-forward model parameters above, the equation for the market value of non-interest-bearing liabilities at time zero is...

$$V_0 = 0.0416 \times (1 - 0.1550) \times 0.2000 \times 1,000,000 \int\limits_0^{\infty} \text{Exp}\left\{\frac{0.1353}{0.1631} + (0.0344 - 0.0908) \times t - \frac{0.1353}{0.1631} \times \text{Exp}\left\{-0.1631 \times t\right\}\right\} \delta t \tag{11}$$

Using the white paper **Solving The Exponential Integral - Excel VBA Toolbox** [4], the solution to Equation (11) above is...

$$V_{0,\infty} = 0.0464 \times (1 - 0.1550) \times 0.20 \times 1,000,000 \times 33.7381 = 264,597 \tag{12}$$

Note that the solution to the exponential integral in Equation (11) above is... [4]

$$\text{Integral value} = \int\limits_0^{\infty} \text{Exp}\left\{\frac{0.1353}{0.1631} + (0.0344 - 0.0908) \times t - \frac{0.1353}{0.1631} \times \text{Exp}\left\{ - 0.1631 \times t\right\}\right\} \delta t$$
$$= \text{EXPINT}(1,\ 0.0908,\ 0.1631,\ 0.1697,\ 0.0344,\ 0{,}0) = 33.7381 \tag{13}$$

References

[1] Gary Schurman, *The Stochastic, Mean-Reverting Short Rate,* May, 2020

[2] Gary Schurman, *Incomplete Gamma Function - Base Equation For A Mean-Reverting Process,* November, 2017

[3] Gary Schurman, *Non-Interest-Bearing Liabilities - Growth Rate is Constant,* March, 2025

[4] Gary Schurman, *Solving The Exponential Integral - Excel VBA Toolbox,* December, 2024